

Inflaton decay in an alpha vacuum

S. Naidu* and R. Holman†

Department of Physics, Carnegie Mellon University, Pittsburgh PA 15213

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Abstract

We study the alpha vacua of de Sitter space by considering the decay rate of the inflaton field coupled to a scalar field placed in an alpha vacuum. We find an *alpha dependent* Bose enhancement relative to the Bunch-Davies vacuum and, surprisingly, no non-renormalizable divergences. We also consider a modified alpha dependent time ordering prescription for the Feynman propagator and show that it leads to an alpha independent result. This result suggests that it may be possible to calculate in any alpha vacuum if we employ the appropriate causality preserving prescription.

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*Electronic address: svn@andrew.cmu.edu

†Electronic address: rh4a@andrew.cmu.edu

I. INTRODUCTION

Field theory in de Sitter space is an area of active interest as the inflationary epoch can be well approximated by de Sitter space. Further, de Sitter space may even be applicable to the current universe as it is known that the expansion is accelerating[1, 2]. Current models of inflation postulate a scalar field rolling to the bottom of its potential as the source of the energy density that drives the expansion and quintessence models attempt to do the same for the present acceleration observed in the universe. Further, in inflation it is the quantum fluctuations in the scalar field that seeds the perturbations responsible for the temperature anisotropy and large scale structure and observations are in agreement with the predictions of inflation[3]. The nature of the de Sitter invariant vacuum is tied to trans-Planckian effects[4] in inflation. For these purposes it is important to investigate the formal construction of field theory in de Sitter space.

Field theory in de Sitter space and the question of possible vacua of free scalar field theory in de Sitter space have been investigated by [5, 6, 7, 8]. One of the surprising results that emerged was a vacuum choice ambiguity that is absent in Minkowski space. They found that in de Sitter space there exists an entire family of de Sitter invariant vacua that can be used to construct a consistent free field theory. This vacuum ambiguity is discussed for general spacetimes by Long and Shore [9] using a wavefunctional formalism. If one insists that the vacuum reduce to the Minkowski vacuum in the limit that the expansion vanishes then a single member of the family is selected and is commonly denoted the Euclidean or Bunch-Davies vacuum [10]. The vacuum choice ambiguity is normally addressed in inflationary calculations by assuming that the vacuum is the distinguished Bunch-Davies vacuum and this leads to successful predictions of the temperature anisotropies. The question of what physics is responsible for the vacuum selection remains to be addressed and a number of papers address the use of the general vacuum with a cut-off to encode trans-Planckian effects[11, 12, 13, 14]. Interacting theories in the general vacua, however, are argued [15, 16, 17] to be inconsistent usually due to the presence of intractable divergences.

Generally the dynamics of inflation are studied treating the inflaton, the scalar field driving inflation, as a classical homogenous field and the gaussian approximation is applied to the quantum fluctuations about this configuration. As mentioned these fluctuations seed the perturbations in the universe. For a review of the quantum treatment of inflationary

dynamics see [18]. Of particular interest to this paper is the question of particle decay in de Sitter space and [19] investigates this questions assuming that the Bunch-Davies vacuum is the relevant one. The same problem is addressed in a somewhat more general context using dynamical renormalization in [20].

Similar studies [15, 16, 17] of interacting theories in general α -vacua, the de Sitter invariant family, are plagued by non-renormalizable divergences that obscure any attempt to probe their properties. Although this is a disturbing feature it does not seem reasonable to simply discard them as they are perfectly valid vacua of the free theory. There have been attempts to address these divergences [21, 22] and in particular we take up a suggestion put forward by Collins and Holman in [23] when we study the problem of particle decay. We find their prescription especially attractive as it yields a vacuum independent result.

In this paper we will address the question of the slowly rolling inflaton decaying into light scalars in a general α -vacuum. We will present a brief introduction to the nature of the vacuum ambiguity of de Sitter space in section (II) and present the notation we will employ. In section (II C) we will present the formalism necessary for calculating in a time dependent background. The interaction lagrangian and the calculation of the decay rate in the Bunch-Davies vacuum follows in section (III). We then present the results in general α -vacua for conformally coupled and minimally coupled in sections (III A) and (III B) respectively. Adopting a prescription for addressing time ordering problems in the α -vacua, we repeat the calculation for the conformally coupled field in section (III C). Finally we end with some concluding remarks regarding a consistent treatment of the α -vacua and their significance.

II. BACKGROUND

The quantization of a free scalar theory in a fixed de Sitter background and the attendant vacuum choice ambiguity is described in [5, 6]. Here we sketch the procedure and describe our conventions. Additional details of the procedure can also be found in [17, 24].

We will find it convenient to work both in cosmic time(1) and conformal time(2) in which the metric is respectively given by

$$\begin{aligned} ds^2 &= dt^2 - e^{2Ht} d\vec{x}^2 \\ &= \frac{d\eta^2 - d\vec{x}^2}{H^2\eta^2} \end{aligned} \tag{1}$$

$$\tag{2}$$

where H , as usual, is the Hubble constant. The domains of the coordinates are $t \in [-\infty, \infty]$ and $\eta \in [-\infty, 0]$ and they are simply related by $\eta = -\frac{e^{-Ht}}{H}$.

A free massive scalar theory satisfies the Klein-Gordon equation

$$[\nabla^2 + m_\Phi^2] \Phi = 0. \quad (3)$$

Since de Sitter space has constant curvature, any coupling to curvature normally denoted by ξR can be absorbed into the mass and hence we take m_Φ to denote an effective mass into which the coupling has been absorbed.

As usual the field is expanded in creation and annihilation operators

$$\Phi(\eta, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[U_k(\eta) e^{i\vec{k}\cdot\vec{x}} a_{\vec{k}} + U_k(\eta)^* e^{-i\vec{k}\cdot\vec{x}} a_{\vec{k}}^\dagger \right] \quad (4)$$

where

$$[a_{\vec{p}}, a_{\vec{q}}^\dagger] = (2\pi)^3 \delta^3(\vec{p} - \vec{q}). \quad (5)$$

Equation (3) implies that the mode functions satisfy the differential equation,

$$\left[\eta^2 \partial_\eta^2 - 2\eta \partial_\eta + \eta^2 k^2 + \frac{m_\Phi^2}{H^2} \right] U_k(\eta) = 0. \quad (6)$$

This equation has Bessel functions as solutions. In particular, if we demand that the modes match the Minkowski massive free scalar modes at short distances then,

$$U_k^E(\eta) = \frac{\sqrt{\pi}}{2} \eta^{3/2} H_\nu^{(2)}(k\eta), \quad (7)$$

where,

$$\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \quad (8)$$

and $H_\nu^{(2)}$ denotes the Hankel function of the second-kind. The normalization of the modes has been fixed by enforcing the canonical commutation relation for the field and its conjugate momentum which leads to a Wronskian condition for the mode functions. We denote the corresponding annihilation operators $a_{\vec{k}}^E$ and the vacuum annihilated by them, commonly referred to as the Bunch-Davies vacuum, as $|E\rangle$. So far, quantization has proceeded much as it does in Minkowski space but we have had to make the reasonable but arbitrary assumption that the modes match Minkowski modes at short distances to fix the vacuum.

A. Alpha vacua

When we relax the Minkowski matching condition we see that any norm preserving linear combination of the mode function and its conjugate will satisfy (6) and preserve the commutation relations. The standard literature addresses this by considering a Bogolubov transformation of the creation-annihilation operators,

$$a_k^\alpha = N_\alpha \left(a_k^E - e^{\alpha^*} a_{-\vec{k}}^{E\dagger} \right), \quad (9)$$

where $\Re[\alpha] < 0$. The vacuum annihilated by all a_k^α is denoted $|\alpha\rangle$. Rewriting (4) in terms of a_k^α we find the corresponding mode functions,

$$U_k^\alpha(\eta) = N_\alpha \left(U_k^E(\eta) + e^\alpha U_k^E(\eta)^* \right). \quad (10)$$

It remains to be shown that this set of vacua parametrized by α – the Bunch-Davies vacuum is recovered at the $\alpha \rightarrow -\infty$ limit – are in fact de Sitter invariant. This can be demonstrated by showing that the two-point functions can be written in terms of the de Sitter invariant distance between the two points. We will construct the two point functions in the next section and address the question briefly while details can be found in [17] and references therein.

B. Green's functions

To compute the diagrams we will eventually encounter in evaluating the decay rate we need to construct the propagators for the free theory. Since the metric we are considering is spatially flat we can write

$$\mathcal{G}^\alpha(x, x') = \langle \alpha | \Phi(x) \Phi(x') | \alpha \rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{x} - \vec{x}')} U_k^\alpha(\eta) U_k^\alpha(\eta')^*. \quad (11)$$

This function depends only on $Z(x, x')$, the de Sitter invariant distance, and hence the de Sitter invariance of the α -vacua. In a momentum representation we have

$$\mathcal{G}_k^\alpha(\eta, \eta') = U_k^\alpha(\eta) U_k^\alpha(\eta')^*, \quad (12)$$

as the momentum space Wightman function.

From the Wightman function we can construct the Feynman propagator following the same procedure as in Minkowski space,

$$G^\alpha(x, x') = i\langle\alpha|T[\Phi(x)\Phi(x')]| \alpha\rangle, \quad (13)$$

$$\begin{aligned} &= \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} G_k^\alpha(\eta, \eta'), \\ G_k^\alpha(\eta, \eta') &= \Theta(\eta - \eta') \mathcal{G}(\eta, \eta') + \Theta(\eta' - \eta) \mathcal{G}(\eta', \eta). \end{aligned} \quad (14)$$

Defined in this manner the Feynman propagator, as would be expected, satisfies the equation of motion with a point source,

$$[\nabla_x^2 + m_\Phi^2] G^\alpha(x, x') = \frac{\delta^{(4)}(x - x')}{\sqrt{-g(x)}}. \quad (15)$$

Although natural, this definition for the Feynman propagator is not the only one leading to a well-defined object, and further, it is perhaps not the most meaningful definition due to subtleties associated with the α -vacua and causality. These issues are addressed in [21, 23, 25] and an α dependent time ordering in the definition of the propagator is suggested. Here we will present the central idea and the details necessary for calculation.

In de Sitter space there is a natural antipode map given in the conformal time coordinatization by $x_A : (\eta, \vec{x}) \rightarrow (-\eta, \vec{x})$. We can then expand

$$\begin{aligned} \mathcal{G}^\alpha(x, x') &= N_\alpha (\mathcal{G}_E(x, x') + e^{\alpha+\alpha^*} G_E(x', x) \\ &\quad + e^\alpha \mathcal{G}_E(x_A, x') + e^{\alpha^*} \mathcal{G}_E(x', x_A)). \end{aligned} \quad (16)$$

This result indicates that ordinary time-ordering will not impose the causality expected. Instead we can define a time-ordering prescription that ensures a causal definition for the Feynman propagator and it leads to a double source propagator,

$$[\nabla_x^2 + m_\Phi^2] G^\alpha(x, x') = A_\alpha \frac{\delta^{(4)}(x - x')}{\sqrt{-g(x)}} + B_\alpha \frac{\delta^{(4)}(x_A - x')}{\sqrt{-g(x_A)}}, \quad (17)$$

where A_α and B_α must be fixed by some prescription while maintaining

$$\lim_{\alpha \rightarrow -\infty} A_\alpha = 1 \quad \text{and} \quad \lim_{\alpha \rightarrow -\infty} B_\alpha = 0 \quad (18)$$

so that the standard time-ordering is recovered in this limit. Doubly sourced propagators have been discussed in [15], but without the modified time ordering they lead to non-renormalizable divergences. We choose the particularly simple scheme $A_\alpha = 1$ and $B_\alpha = 0$.

This leads to a singly sourced propagator and in the path integral formulation all internal lines are naturally converted to the Bunch-Davies propagator. This conveniently takes care of the divergences that plague calculations in the general α -vacua. We should note however that the two-point functions are still α dependent.

C. Schwinger-Keldysh formalism

Since de Sitter space presents a time dependent background it is not meaningful to calculate S-matrix elements. The absence of a global time like Killing vector does not allow us to relate asymptotic *in* and *out* states and at any rate we mean to calculate the matrix element between time evolving states at a given time. The Schwinger-Keldysh [26, 27, 28, 29] formalism addresses these issues by calculating the evolution of matrix elements from given initial conditions to some later time. The application of the formalism to de Sitter space is discussed in [17, 19].

We can summarize the results by considering the evolution of the density matrix. If we define the evolution operator so that

$$i\frac{\partial}{\partial\eta}U_I(\eta,\eta_0)=H_I U_I(\eta,\eta_0), \quad (19)$$

and we take the full Hamiltonian to be given by

$$H=H_0+\Theta(\eta-\eta_0)H_I. \quad (20)$$

We define initial conditions at η_0 where $\rho(\eta_0)=\rho_0$. Then we can calculate the time-dependent expectation value of a general operator,

$$\langle\mathcal{O}\rangle(\eta)=\frac{\text{Tr}[U_I(-\infty,0)U_I(0,\eta)\mathcal{O}U_I(\eta,-\infty)\rho_0]}{[Tr][U_I(-\infty,0)U_I(0,-\infty)\rho_0]} \quad (21)$$

The numerator of this expression represents a closed time contour (fig. 1) running from the infinite past with given initial conditions, enclosing the operator inserted at η , the infinite future and finally running back to the infinite past.

The calculation is implemented formally by considering fields that live on the forward contour and a separate set of fields that live on the backwards contour, labelled respectively by '+' and '-'. Time ordering is then implemented so that '-' fields always appear after

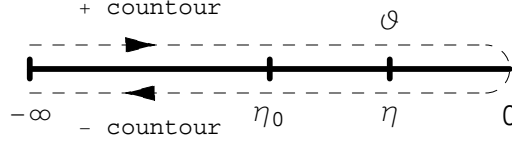


FIG. 1: The closed time contour that appears in the evolution of operators over a finite time interval. Separate copies of the field appear on the forward running and backward running portions of the contour. Here the contour is labelled in conformal coordinates.

'+' fields and that time runs backward on the backward contour. The respective Green's functions are given by,

$$G_k^{(++)}(\eta, \eta') = \Theta(\eta - \eta') \mathcal{G}_k(\eta, \eta') + \Theta(\eta' - \eta) \mathcal{G}_k(\eta', \eta) \quad (22)$$

$$G_k^{(--)}(\eta, \eta') = \Theta(\eta' - \eta) \mathcal{G}_k(\eta, \eta') + \Theta(\eta - \eta') \mathcal{G}_k(\eta', \eta) \quad (23)$$

$$G_k^{(-+)}(\eta, \eta') = \mathcal{G}_k(\eta, \eta') \quad (24)$$

$$G_k^{(+-)}(\eta, \eta') = \mathcal{G}_k(\eta', \eta) \quad (25)$$

where we notice that G^{++} is the usual Feynman propagator. Further, due to the doubling of fields, we have '+' and '-' vertices corresponding to each of the vertices in the original interaction lagrangian. Further details of this formal procedure will become apparent when we present the calculation of the decay rate.

III. INTERACTION LAGRANGIAN

We now describe the model we will use to study inflaton decay [19] which consists of the inflaton field coupled to a massless scalar field, σ via a three point interaction. We anticipate the requirements of the Schwinger-Keldysh formalism and include both '+' and '-' fields in the lagrangian,

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}(\partial_\mu \Phi^+ \partial^\mu \Phi^+ + m_\Phi^2 \Phi^{+2} + \partial_\mu \sigma^+ \partial^\mu \sigma^+ + \xi_\sigma R \sigma^{+2}) \\ & + \frac{1}{2} g \Phi^+ \sigma^{+2} + h(t) \Phi \\ & - [+ \rightarrow -] \end{aligned} \quad (26)$$

where the negative sign appearing before $\mathcal{L}(\Phi^-, \sigma^-)$ is a result of the time reversal on the negatively directed time contour,

$$\begin{aligned} S_I &= - \int_{-\infty}^0 H_I(\Phi^+, \sigma^+) d\eta - \int_0^{-\infty} H_I(\Phi^-, \sigma^-) d\eta \\ &= - \int_{-\infty}^0 [H_I(\Phi^+, \sigma^+) - H_I(\Phi^-, \sigma^-)] d\eta. \end{aligned} \quad (27)$$

The interaction term is apparent, but the magnetic term involving $h(t)$ requires some explanation. Since we are primarily interested in the decay of Φ into σ we need not be concerned with the details of the potential for Φ as long as the field is rolling slowly with respect to the decay rate into σ . So instead of including an explicit, but arbitrary, potential we simply include a magnetic (or source) term to set initial conditions such that Φ is 'slowly rolling'. In addition $h(t)\Phi$ term is required for the renormalization of tadpole diagrams[19].

As usual the inflaton field is separated into a homogenous term and scalar fluctuations,

$$\Phi^\pm(\vec{x}, t) = \phi(t) + \varphi^\pm(\vec{x}, t) \quad \text{where} \quad \langle \Phi^\pm \rangle = \phi(t). \quad (28)$$

This implies the tadpole condition

$$\langle \varphi^\pm(\vec{x}, t) \rangle = 0 \quad (29)$$

from which we obtain the equation of motion. In addition we also enforce $\langle \sigma(\vec{x}, t) \rangle = 0$ which ensures that the σ field does not acquire an expectation value.

Rewriting the action in terms of the shifted fields we find

$$\begin{aligned} S &= \int d^4x \sqrt{-g} \left\{ \mathcal{L}_0(\varphi^+) + \mathcal{L}_0(\sigma^+) \right. \\ &\quad + \varphi^+ [-\ddot{\phi} - 3H\dot{\phi} - m_\Phi^2 \phi] \\ &\quad + \frac{g}{2}(\phi + \varphi^+)(\sigma^+)^2 + h\varphi^+ \\ &\quad \left. - [+ \rightarrow -] \right\}, \end{aligned} \quad (30)$$

where \mathcal{L}_0 denotes the lagrangian of the corresponding free theories. Now evaluating the

tadpole condition

$$\begin{aligned}
\langle \varphi^+(\vec{x}, \tau) \rangle &= \int \mathcal{D}\varphi^\pm \mathcal{D}\sigma^\pm \varphi^+(\vec{x}, \tau) e^{iS} \\
&= \int d^3y dt \sqrt{-g} i \left[\ddot{\phi} + 3H\dot{\phi} + m_\Phi^2 \phi - \frac{g}{2} \langle \sigma^2 \rangle(t) - h(t) \right. \\
&\quad \left. - i \frac{g^2}{4\pi^2} \int_{-\infty}^t dt' \sqrt{-g} K(t, t') \phi(t') \right] \times \\
&\quad (\langle \varphi^+(\vec{x}, \tau) \varphi^+(\vec{y}, t) \rangle - \langle \varphi^+(\vec{x}, \tau) \varphi^-(\vec{y}, t) \rangle) \\
&\quad + \mathcal{O}(g^3)
\end{aligned} \tag{31}$$

where we have defined

$$K(t, t') \equiv \int_0^\infty dk k^2 (G_k(t', t)^2 - G_k(t, t')^2). \tag{32}$$

To enforce the tadpole condition we must require that the coefficient of $(\langle \varphi^+(\vec{x}, \tau) \varphi^+(\vec{y}, t) \rangle - \langle \varphi^+(\vec{x}, \tau) \varphi^-(\vec{y}, t) \rangle)$ vanish and this yields the equation of motion for $\phi(t)$. First we must take care of the $\langle \sigma^2 \rangle$ term which we simply absorb into $h(t)$ defining

$$h_\sigma(t) = h(t) + \frac{g}{2} \langle \sigma^2 \rangle(t). \tag{33}$$

The inhomogenous term does not enter the expression for the decay rate and so is not of concern here. Its main function is to act as a Lagrange multiplier to enforce the initial conditions at $t = 0$,

$$\phi(0) = \phi_i \tag{34}$$

$$\dot{\phi}(0) = 0 \tag{35}$$

$$\tag{36}$$

It turns out that the kernel $K(t, t')$ can be cast in convolution form $K(t - t')$ and so the equation can be solved by Laplace transforming $\phi(t)$. In general we define

$$\tilde{f}(s) = \mathcal{L}\{f(t)\} \equiv \int_0^\infty dt e^{-st} f(t). \tag{37}$$

The exact form of the kernel depends on the nature of the coupling of the σ field to curvature and its mass. However, generally the kernel integral possesses a logarithmic UV divergence which can be absorbed into a mass redefinition. In the Bunch-Davies vacuum the renormalized mass is given by

$$m_{\Phi, R}^2 = m_\Phi^2 + \frac{g^2}{16\pi^2} (\ln \epsilon + 1) \tag{38}$$

The details of Laplace transforming the equation of motion also depend on properties of the σ field but as an illustrative example for the conformally coupled massless case in the Bunch-Davies vacuum [19] it is found that

$$s^2\tilde{\phi}(s) - s\phi_i + 3H(s\tilde{\phi}(s) - \phi_i) + m_{\Phi,R}^2\tilde{\phi}(s) + \frac{g^2}{16\pi^2}\Sigma(s)\tilde{\phi}(s) = \tilde{h}_\sigma(s) \quad (39)$$

$$\Sigma(s) = -s\mathcal{L}\{\ln(1 - e^{-Ht}) + e^{-Ht}\}. \quad (40)$$

Solving this algebraic equation for $\tilde{\phi}(s)$ and taking the inverse Laplace transform the decay rate can be identified from resulting solution for $\phi(t)$. In the particular example we are discussing the decay rate is found to be [19]

$$\Gamma = \frac{g^2}{16\pi^2} \frac{\Im[\Sigma(s_0^+)]}{M_\Phi} = \frac{g^2}{32\pi M_\Phi} \tanh\left(\frac{\beta_H M_\Phi}{2}\right) \quad (41)$$

where $\beta_H = \frac{2\pi}{H}$ and to this order

$$M_\Phi^2 = m_\Phi^2 - \frac{9H^2}{4} \quad (42)$$

$$s_0^+ = -\frac{3H}{2} + iM_\Phi. \quad (43)$$

The same general procedure is employed in the following sections. However, the mode functions and hence the Green's functions vary for different theories (conformally coupled/minimally coupled) and α -vacua. This implies that the kernel acquires new features and, in the case of a minimally coupled field, infrared divergences. However, after some work the decay rate can be extracted in the same fashion from the solution to the equation of motion. These results are presented in the next two sections.

A. Conformally coupled massless σ

Specializing to the conformally coupled massless case, the calculation for general α proceeds in the same manner as detailed above for the Bunch-Davies vacuum, except the appropriate α dependent Green's functions are employed in computing the diagram. Defining

$$E_\alpha = \frac{1 + e^{\alpha+\alpha^*}}{1 - e^{\alpha+\alpha^*}} \quad (44)$$

$$D_\alpha = \frac{e^\alpha + e^{\alpha^*}}{1 + e^{\alpha+\alpha^*}} \quad (45)$$

we can write the α dependent mass renormalization and self-energy as

$$m_{\Phi,R}^2 = m_{\Phi}^2 + E_{\alpha} \frac{g^2}{16\pi^2} (\ln \epsilon + 1) \quad (46)$$

$$\Sigma(s) = E_{\alpha} (\Sigma_E(s) + D_{\alpha} \tilde{\chi}(s)) \quad (47)$$

where

$$\tilde{\chi}(s) = \frac{H}{s + 2H} - \frac{H}{s + H} \quad (48)$$

is a non-multiplicative correction to the self-energy. It can be seen that the mass renormalization differs from the euclidean case only multiplicatively and it should be noted that as $\alpha \rightarrow -\infty$ (the Bunch-Davies limit) the multiplicative factor $E_{\alpha} \rightarrow 1$. Although the modification to the self-energy is not purely multiplicative in general, the imaginary part *is* only multiplicatively modified.

$$\Gamma = E_{\alpha} \frac{g^2}{32\pi m_{\Phi}} \tanh\left(\frac{\beta_H M_{\Phi}}{2}\right) = E_{\alpha} \Gamma_E \quad (49)$$

Rewriting the multiplicative factor in a suggestive form

$$E_{\alpha} = 1 + 2(e^{-(\alpha+\alpha^*)} - 1)^{-1} \quad (50)$$

we can interpret the decay rate as being Bose enhanced with respect to the Bunch-Davies rate. It should be noted that the Bunch-Davies rate is unique in that it is the minimum rate since $E_{\alpha} \geq 1$.

B. Minimally coupled massless σ

The mode solution for the minimally coupled scalar in the Bunch-Davies vacuum is given by

$$U_k^E(\eta) = (i - k\eta) \frac{e^{-ik\eta}}{\sqrt{2k^{3/2}}} \quad (51)$$

and from this we can construct the general mode functions $U_k^{\alpha}(\eta)$. Then the Green's functions follow and once again the procedure as outlined above can be invoked. However, new complications arise, the first of which is an infrared divergence presumably due to the massless minimal coupling. The divergence can be regulated by imposing a lower cut-off on the loop integral. Further, the resulting equation of motion after Laplace transforming is no longer algebraic but rather a first order differential equation of the form,

$$\left[p_0(s) + \lambda \left(p_1(s) + q(s) \frac{d}{ds} \right) \right] \tilde{\phi}(s) = f(s) \quad (52)$$

Solving this equation, order by order in λ ,

$$\tilde{\phi}(s) = \tilde{\phi}_0(s) + \lambda \tilde{\phi}_1(s) + \mathcal{O}(\lambda^2) \quad (53)$$

$$\tilde{\phi}_0(s) = \frac{f(s)}{p_0(s)} \quad (54)$$

$$\tilde{\phi}_1(s) = -\frac{\tilde{\phi}_0(s)}{p_0(s)} \left[p_1(s) + q(s) \frac{d \ln \tilde{\phi}_0}{ds} \right] \quad (55)$$

leads to the result

$$\tilde{\phi}(s) = \frac{f(s)}{p_0(s) + \lambda \left[p_1(s) + q(s) \frac{d}{ds} \ln \frac{f(s)}{p_0(s)} \right]} + \mathcal{O}(\lambda^2), \quad (56)$$

This is of the same form as the conformally coupled case, at least to $\mathcal{O}(g^4)$. The decay rate, once again, acquires a purely multiplicatively enhancement, and we find,

$$\Gamma = E_\alpha \frac{g^2}{32\pi M_\Phi} \left[\left(1 + \frac{4H^2}{m_\Phi^2} \right) \tanh \left(\frac{\beta_H M_\Phi}{2} \right) + \frac{8H^3 M_\Phi}{\pi m_\Phi^4} \right] \quad (57)$$

C. Alpha Time Ordering

The absence of divergences in the preceding calculation of the decay rate in an alpha vacuum is unusual considering that previous investigations [16, 17, 30] seem to indicate that non-renormalizable divergences inevitably appear in alpha vacuum calculations. The significance of these divergences is still unclear but in certain cases they can be traced to pinched singularities in the propagator.

As discussed in section (II B) it is possible to define an α -dependent time ordering prescription that resolves the pinched singularities appearing in the Feynman propagator. A related procedure is discussed in [22] where non-local interactions are introduced to absorb the divergences that arise while preserving the locality of the theory. In fact, the inclusion of a non-local (point/anti-pode) interactions that preserve the locality of the theory in the sense that commutators vanish outside the light cone was discussed in [23, 25]. So for this section we will generalize the interaction lagrangian to include these antipodal interactions.

Before we present the new interaction lagrangian, it is important to note that the main result of the time ordering prescription together with our choice for the arbitrary constants $A_\alpha = 1$ and $B_\alpha = 0$ is that internal propagators are converted to the Bunch-Davies propagator and all the α -dependence is restricted to the external legs. This is distinct from a

field redefinition in that we hold the interaction term fixed when we change the ordering prescription. A field redefinition would demand the interaction term undergo the corresponding transformation. This point is discussed in [25] where it is clarified that a redefinition of the kinetic term to make it once again appear local would lead to a non-local interaction term. The appearance of euclidean propagators on internal lines indicates that the only divergences that will appear are the ones encountered in the Bunch-Davies vacuum which can be renormalized via the usual mass and (if relevant) wave function and coupling constant renormalization.

We write the new interaction lagrangian that includes antipodal interactions as

$$\mathcal{L}_I = \frac{1}{2} e^{3Ht} g \sum_{i,j \in \{\mathcal{P}, \mathcal{A}\}} \lambda_{ij} \Phi(x) \sigma(x_i) \sigma(x_j)$$

where we take $x_{\mathcal{P}} = (\eta, \vec{x})$ and $x_{\mathcal{A}} = (-\eta, \vec{x})$ and λ_{ij} is a dimensionless matrix describing the relative strengths of the interaction terms. To fix the spurious freedom in the definition of λ we impose

$$\lambda_{\mathcal{P}\mathcal{P}} = 1 \tag{58}$$

$$\lambda = \lambda^T \tag{59}$$

This particular choice of conditions ensures that the earlier result is recovered when all the free components of λ approach zero.

Now there are a sum of diagrams to be evaluated resulting in the kernel given by

$$K(\eta, \eta') = \sum_{ii'jj'} \int_0^\infty k^2 dk \lambda_{ij} \lambda_{i'j'} \{ G_k^{++}(\eta_i, \eta'_{i'}) G_k^{++}(\eta_j, \eta'_{j'}) - G_k^{+-}(\eta_i, \eta'_{i'}) G_k^{+-}(\eta_j, \eta'_{j'}) \}$$

For the massless conformally coupled case, $\nu = \frac{1}{2}$, only UV divergences appear in the integral

which can be regulated as was done previously with $e^{ika} \rightarrow e^{ik(a+i\epsilon)}$. In this case we find

$$\begin{aligned}
\frac{1}{\eta^2 \eta'^2} K(\eta, \eta') = & \\
& \lambda_{\mathcal{PP}}^2 \Theta(\eta - \eta') \frac{4i(\eta - \eta')}{\epsilon^2 + 4(\eta - \eta')^2} \\
& - \lambda_{\mathcal{PP}} \lambda_{\mathcal{AA}} \frac{4i(\eta + \eta')}{\epsilon^2 + 4(\eta + \eta')^2} \\
& + \lambda_{\mathcal{AA}}^2 \Theta(\eta' - \eta) \frac{4i(\eta' - \eta)}{\epsilon^2 + 4(\eta' - \eta)^2} \\
& + 4\lambda_{\mathcal{PA}} \left\{ \lambda_{\mathcal{PP}} \left[\frac{1}{2} \left(\frac{1}{\epsilon - 2i\eta} - \frac{1}{\epsilon + 2i\eta'} \right) - \Theta(\eta - \eta') \left(\frac{1}{\epsilon - 2i\eta} - \frac{1}{\epsilon - 2i\eta'} \right) \right] \right. \\
& + \lambda_{\mathcal{AA}} \left[\frac{1}{2} \left(\frac{1}{\epsilon - 2i\eta'} - \frac{1}{\epsilon + 2i\eta} \right) - \Theta(\eta - \eta') \left(\frac{1}{\epsilon - 2i\eta'} - \frac{1}{\epsilon - 2i\eta} \right) \right] \\
& + \lambda_{\mathcal{PA}} \left[\frac{1}{\epsilon} + \Theta(\eta - \eta') \frac{4i(\eta - \eta')}{\epsilon^2 + 4(\eta - \eta')^2} \right. \\
& \left. \left. - \frac{1}{2} \left(\frac{1}{\epsilon - 2i(\eta + \eta')} + \frac{1}{\epsilon - 2i(\eta - \eta')} \right) \right] \right\}
\end{aligned} \tag{60}$$

First of all we should note that the non-local interactions mixing point/antipode fields – the $\lambda_{\mathcal{PA}}$ terms – give rise to a linear divergence in the kernel. More problematically, all terms involving the antipodal field give rise to terms of the form

$$\int_{\eta}^0 d\eta' \phi(\eta') f(\eta', \eta)$$

in the equation of motion which manifestly violate causality, atleast in our framework with adiabatic switching in the infinite past. Boundary conditions of this form have been considered in the context of elliptic de Sitter space, see [31] and references therein. Although the theory is local in the sense that commutators vanish outside the light cone, we are led to conclude that the resulting theory is not causal in our framework since we cannot define a useful initial value boundary condition. So it appears that the only meaningful choice for the interaction matrix is

$$\lambda_{\mathcal{AA}} = \lambda_{\mathcal{PA}} = 0.$$

With this choice for the interaction the prescription for modifying the propagators recovers the euclidean result for the decay rate. Further, in [23] it was noted that euclidean self energy is also recovered. This, of course, is not surprising considering that in essence the prescription sets all to internal propagators to the euclidean propagator. This suggests that proper attention to causality – which motivated the propagator redefinition – removes any

vacuum ambiguity in an interacting theory. It provides an additional motivation for calculating in the euclidean vacuum, namely, it is the vacuum that yields the natural feynman propagator since its definition depends of enforcing causality. However, we can still choose to calculate in any vacuum but physical results will be independent of our choice.

IV. CONCLUSIONS

In this paper we have considered the decay of the inflaton into light scalars placed in a general de Sitter invariant α -vacuum. The result for the commonly considered Bunch-Davies vacuum, [19], yields a decay rate that corresponds to decay in Minkowski space at a finite temperature, $\beta_H = 2\pi/H$. For a general α -vacuum we find a further enhancement that is analogous to Bose enhancement seen in stimulated emission with the enhancement factor given in eq.(50). We find that this is the case for both conformally and minimally coupled scalars. This interpretation is compelling considering that when α -vacua are projected on the Hilbert space constructed from the Bunch-Davies vacuum they correpond to states with constant occupation of all modes which would lead to stimulated decay. Of course, this still singles out the Bunch-Davies vacuum as it is relative to the Bunch-Davies rate that the rate for general α are enhanced.

We have also considered a recently proposed prescription for treating the α -vacua that seeks to address the proper time-ordering of the Feynman propagator in a general α -vacuum. The prescription leads to the internal lines of diagrams being converted to the Bunch-Davies propagator independent of α . This leads to a self-energy, cacculated in [23], and decay rate, as discussed in this paper, that are independent of α . Further, the only divergences that can arise from loops are those that are encountered in the Bunch-Davies vacuum and hence the theory is renormalizable for all α . This is an appealing result as it leads to an α independent scheme for calculation. However, there is still α dependence in the external legs which will appear for example in the two-point function relevant to inflation. This seems to be related to the manner of initial conditions and is question that we are still investigating.

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